Appendix: Semiclassical computation of ground state energy of tilted supersymmetric double–well potential

Eigenlevels of the Schrödinger equation

$$\left[-\frac{1}{2}\partial^2 + \frac{g}{2}\left(y^2 - \frac{1}{4g}\right)^2 - \sqrt{g}y\right]\chi(y) = \nu\chi(y) \quad \text{(A.1)}$$

can be studied in the limit of $g \ll 1$ via semiclassical approximation. Proper quantization condition taking into account both perturbative and non-perturbative corrections in small g can be derived via uniform WKB[1]. Non-perturbative corrections to zeroth energy level (where they are the only ones), as well as to higher levels (fully determining the LL level splitting) can be found in the following way.

Near the minimum $y_\pm = y \mp \frac{1}{2\sqrt{g}}$ potential is quadratic ($\nu_+ \equiv \nu$, $\nu_- \equiv \nu - 1$)

$$\left[-\partial^2 + y_\pm^2 - (2\nu_\pm + 1)\right]\chi(y_\pm) \approx 0, \quad |y_\pm| \ll \frac{1}{\sqrt{g}}.$$

Decaying at $y \to +\infty$ solution is given by Hermite function $\psi_\nu^{\rm osc}(y)$ which asymptotes

$$\psi_\nu^{\rm osc} \sim \begin{cases} (2y)^\nu e^{-\frac{y^2}{2}}, & y \to +\infty, \\ \cos\pi\nu(-2y)^\nu e^{-\frac{y^2}{2}} + \dfrac{\sqrt{\pi}e^{\frac{y^2}{2}}}{\Gamma(-\nu)(-y)^{\nu+1}}, & y \to -\infty. \end{cases}$$

We have to match $\psi_\nu^{\rm osc}$ with WKB solution, valid under the potential hump $|y| \sim \frac{\rm const}{\sqrt{g}}$

$$\chi = C_+ \frac{\exp\int_0^y |k(z)|dz}{\sqrt{|k(y)|}} + C_- \frac{\exp-\int_0^y |k(z)|dz}{\sqrt{|k(y)|}}, \quad \text{(A.2)}$$

$$|k(y)|^2 = g\left(y^2 - \frac{1}{4g}\right)^2 - 2\sqrt{g}y - 2\nu. \quad \text{(A.3)}$$

To this end, we expand semiclassical action at $|y| \lesssim \frac{1}{2\sqrt{g}}$

$$S(y) = \frac{1}{8g}\int_0^{2\sqrt{g}y} \sqrt{(1-z^2)^2 - 16g(z+2\nu)}\,dz \sim$$
$$\sim \left[\frac{1}{8g}\left(z - \frac{z^3}{3}\right) + \frac{1}{2}\ln\left|1-z^2\right| + \nu\ln\left|\frac{1-z}{1+z}\right|\right]\Bigg|_0^{2\sqrt{g}y}$$

and near the minimum $|y_\pm| \ll \frac{1}{\sqrt{g}}$

$$S(y) \sim \frac{1}{12g} - \frac{y_+^2}{2} + \frac{1}{2}\ln\left[-4\sqrt{g}y_+\right] + \nu\ln\left[-\sqrt{g}y_+\right],$$
$$\sim -\frac{1}{12g} + \frac{y_-^2}{2} + \frac{1}{2}\ln\left[4\sqrt{g}y_-\right] - \nu\ln\left[\sqrt{g}y_-\right]$$

where we neglected terms $\mathcal{O}(y_\pm^3)$. Matching (A.2) with asymptotics of Hermite functions we derive the quantization condition

$$\left(\frac{g}{2}\right)^{2\nu}\Gamma(\nu)\Gamma(1+\nu)\tan^2\pi\nu = \frac{\pi}{2}e^{-\frac{1}{3g}}, \quad \text{(A.4)}$$

which for $\nu \to 0$ becomes $\nu_0 = e^{-\frac{1}{3g}}/2\pi$ and for $\nu \to \mathbb{N}_+$ gives energy level splittings due to inter-well tunnelling. These results have been derived via instanton technique in Refs. 2 and 3 and in similar way in Ref. 4.

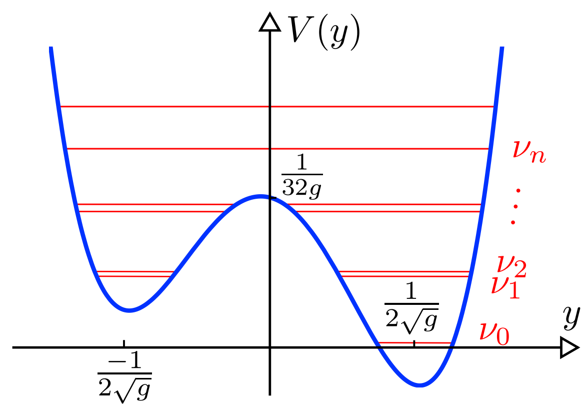

FIG. 1: Tilted supersymmetric double–well potential (A.1) and its energy levels ν_n with coupling constant $g = 2^{-6}$.

Appendix: Supplementary Figure

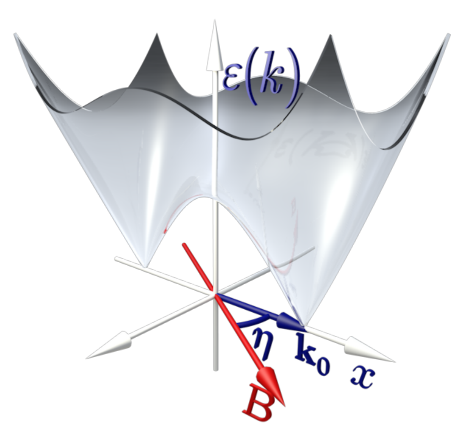

FIG. 2: The orientation of Weyl spectrum and magnetic field.

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