PROBLEM SET #6
"Periodic potential"
due on 5/17/2021

Physics–172 / Applied Physics–272
Introduction to Solid State Physics
Spring quarter, 2021

Instructor: *Aharon Kapitulnik* — aharonk@stanford.edu
Class Assistant: *David Saykin* — saykind@stanford.edu
Departments of Applied Physics and of Physics

Comment. *Note that since these lecture notes are part of a manuscript, much of the language used here follows that of a book, with reference to chapters, sections, etc. In addition, some material in the notes may be more advanced, or will not directly be related to discussions in class. Please view these parts as stimulating extensions, where you are welcome to contact us about questions and relevant references.*

Problem 6.1 (Nearly free electrons— **4 points**)**.** Consider one–dimensional periodic potential of triangle form as small perturbation.

$$\hat{\mathcal{H}} = \frac{\hat{p}^2}{2m} + \frac{\hbar^2 q^2}{m} v_a(x), \qquad v_a(x) = \sum_{n=-\infty}^{\infty} \left[1 - \frac{4}{a}|x - na|\right] \theta\left(2 - \frac{4}{a}|x - na|\right).$$

How does small periodic potential change spectrum of free electrons? Find the width of energy band gaps Δ_n up to the first order in potential strength $q^2 \ll 1/a^2$.

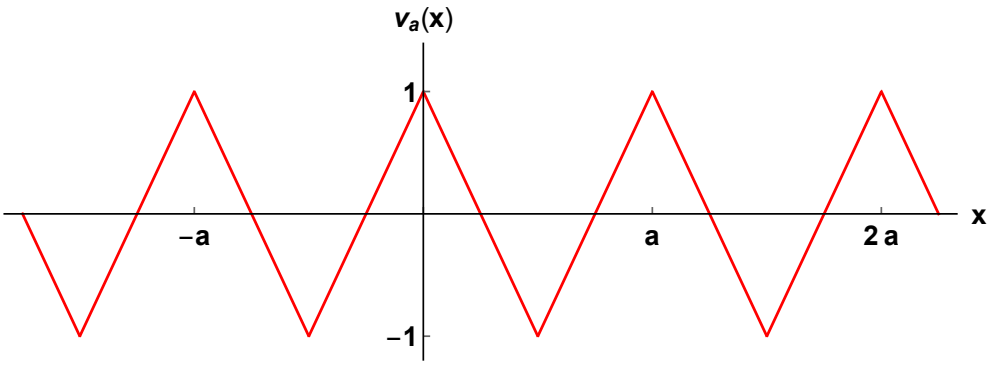

Figure 1: Triangle periodic potential.

Problem 6.2 (δ–Kronig–Penny model— **7 points**)**.** Consider a one dimensional crystal for which the potential is a periodic array of delta-functions,

$$V(x) = \frac{\hbar^2 q}{m} \sum_{n=-\infty}^{\infty} \delta(x - na).$$

where a is the lattice constant.

1. (Exact solution — **4 points**) Compute the energies for this model. Draw a picture of the band structure for positive and negative values of q.

2. (Tight–binding limit — **3 points**) If qa is large and negative, there is a tightly bound Wannier state $w_n(x)$ associated with each delta function. Find zero–order Wannier functions and calculate hopping amplitude $t = \gamma - \alpha\beta$. Also, deduce value of t from exact solution and compare the two.

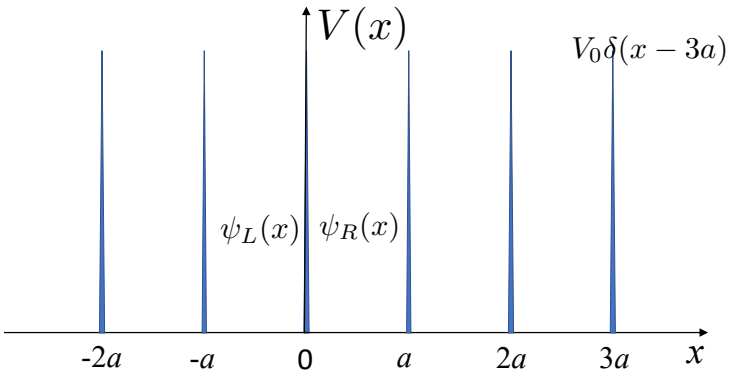

Figure 2: The δ-function Kronig-Penny Model