PROBLEM SET #8

"Graphene"

due on 5/31/2021

Physics–172 / Applied Physics–272

Introduction to Solid State Physics

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Problem 8.1 (Phonons — **8 points**). Consider one–dimensional chain of atoms connected through springs with stiffness K. There are two alternating types of atoms with masses m_1 and m_2, distance between the atoms of the same mass is a (see Figure 1).

1. (**4 points**) Find dispersion relation $\omega(k)$ for the frequency of oscillations of atoms in one–dimensional chain
2. (**1 point**) Find sound velocity.
3. (**3 points**) What happens in the limit $m_1 = m_2$?

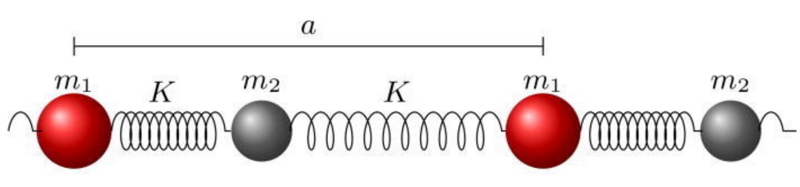

Figure 1: Model of 1D crystal.

Problem 8.2 (Graphene photodetector — **14 points**). The goal of this question is to evaluate graphene performance as a bolometer–type photodetector. Graphene is a two-dimensional carbon monolayer in which low-energy electronic states are well described by a linear band dispersion $E(\mathbf{p}) = v|\mathbf{p}|$ for electrons and $E(\mathbf{p}) = -v|\mathbf{p}|$ for holes. The velocity value is $v = 10^6$ m/s.

Consider a graphene monolayer sample of size 1 micron by 1 micron that absorbs one photon of energy 2.7 eV (blue light). Taking initial temperature to be $T = 2$ K, find the change in electron temperature resulting from photon absorption. Assume that the photon energy is shared among the electron degrees of freedom and ignore the effects of electron–lattice cooling. Please provide your answer in kelvin.

1. (**6 points**) Assume that graphene is undoped, i.e. Fermi level is exactly at the Dirac point.
2. (**7 points**) Now suppose that a small number of carriers are added to the band at a total density of $n = 10^{12}$ cm^{-2}. Repeat the analysis of part 1 to find the change in electron temperature after a single photon absorption.

Could your answers be used as reasonable estimates for a real graphene? Explain (**1 point**).

Problem 8.3 (Landau levels in graphene — **11 points**). Solve stationary Schrödinger problem[1] for electrons in graphene in the presence of the magnetic field $\mathbf{B} = B\hat{z}$.

$$\hat{\mathcal{H}} = v\left(\hat{\boldsymbol{\sigma}}, \hat{\mathbf{p}} + \frac{|e|}{c}\mathbf{A}\right), \qquad \mathbf{A} = (-By, 0, 0).$$

1. (**8 point**) Find energies ε_n and eigenvectors $\psi_{n,k}$. You can use harmonic oscillator functions (a.k.a normalized Hermite functions) $\psi_n^{\text{osc}}(x)$ in your answer.

$$\psi_n^{\text{osc}}(x) = \frac{e^{-x^2/2}H_n(x)}{\sqrt{2^n n!\sqrt{\pi}}}.$$

   Pay attention to zeroth Landau level. Which eigenvector does it correspond to?

2. (**3 points**) How would your answer change for electrons in valley K'?

[1] In case you are interested how to justify the Hamiltonian above, read on the Peierls substitution.