PROBLEM SET #3

"Magnetism of electron gas"

due on 4/26/2021

Physics–172 / Applied Physics–272

Introduction to Solid State Physics

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Comment. *Note that since these lecture notes are part of a manuscript, much of the language used here follows that of a book, with reference to chapters, sections, etc. In addition, some material in the notes may be more advanced, or will not directly be related to discussions in class. Please view these parts as stimulating extensions, where you are welcome to contact us about questions and relevant references.*

Problem 3.1 (Sommerfeld expansion in 2D — **3 points**)**.** Applying the magnetic field in the z-direction leaves the x-y plane to respond to the field as a *two-dimensional electron gas* (2DEG).

1. (**1 point**) Derive the relation between electron density n and Fermi wavevector k_F and calculate the density of states $g(\epsilon)$ of the free electron system in two-dimensions.

2. (**1 point**) Show that except for the $T = 0$ term, all other terms in the low temperature expansion (a.k.a. the Sommerfeld expansion) of the electron density $n(\mu, T)$ vanish.

3. (**1 point**) Show that

$$\mu + T \ln\left(1 + e^{-\mu/T}\right) = \epsilon_F$$

   Explain how this formula reconciles with Sommerfeld expansion you've derived above.

Problem 3.2 (Pauli paramagnetism — **3 points**)**.** Let's consider electron gas that interacts with magnetic field via Zeeman mechanism only (orbital field could be considered separately)

$$\hat{\mathcal{H}} = \frac{\hat{p}^2}{2m} - m_z H, \qquad \hat{m}_z = -2\mu_B \hat{s}_z = -\mu_B \hat{\sigma}_z, \quad \mu_B = \frac{|e|\hbar}{2mc}.$$

Compute magnetic susceptibility in the $T \ll E_F$ limit and small enough magnetic field (what it should be compared to for $D = 2$? for $D = 3$?). For the sake of exercise, we ask you to do the calculation using canonical ensemble free energy $F = \Omega + \mu N$.

$$\mathbf{m} = -\left(\frac{\partial F}{\partial \mathbf{H}}\right)_{N,V,T}. \tag{3.2.1}$$

Problem 3.3 (Paramagnetic heat capacity — **2 points**)**.** Without any calculation, estimate what you expect should be the correction to the specific heat of an electron system at low temperatures due to Zeeman effect.

Problem 3.4 (Langevin Paramagnetism — **3 points**). Magnetic properties of solids are different from those of free atoms because they come from electron quasiparticles rather than actual electrons. There are however known cases of paramagnetic response of the Langevin (i.e. classical) type in solids with rare–earth ions (such as Nd^{3+}, Gd^{3+}, Dy^{3+}, Er^{3+}). In these metals, magnetism is due to $4f$ valence electron, which are screened by electrons on fifth and sixth energy levels, hence behave like electrons in free atoms.

Magnetic energy of the electrons in a single free atom is given by Zeeman term (and orbital field, which is negligible if total angular moment $J \neq 0$).

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 - \hat{\mathbf{m}}_{\mathrm{at}} \cdot \mathbf{H},$$

Here $\hat{\mathcal{H}}_0$ is field independent part of the energy, and $\hat{\mathbf{m}}_{\mathrm{at}} = -\mu_B(\hat{\mathbf{L}} + 2\hat{\mathbf{S}})$ is an operator of the total magnetic moment of the atom. Derive expression for the magnetic susceptibility in terms of magnitude of the total angular moment $\hat{\mathbf{J}} = \hat{\mathbf{L}} + \hat{\mathbf{S}}$ and Lande g–factor. Plug in values for the ${}^4I_{9/2}$ electronic term (which corresponds to $4f$ shell of Nd^{3+})