PROBLEM SET #5
"Electron transport"
due on 5/10/2021

Physics–172 / Applied Physics–272
Introduction to Solid State Physics
Spring quarter, 2021

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Comment. *Note that since these lecture notes are part of a manuscript, much of the language used here follows that of a book, with reference to chapters, sections, etc. In addition, some material in the notes may be more advanced, or will not directly be related to discussions in class. Please view these parts as stimulating extensions, where you are welcome to contact us about questions and relevant references.*

Problem 5.1 (Friedel oscillations — **4 points**). Calculate local density of states (LDOS)

$$\rho(x) = 2 \sum_{k<k_F} |\psi_k(x)|^2 ,$$

where $\psi_k(x)$ are one–particle wavefunctions with wavevector k and factor 2 comes from spin, for the system of

1. (**1 point**) Fermions living on a half–line $x > 0$ with impenetrable wall $\psi(+0) = 0$.
2. (**3 points**) Fermions on a line with scattering center $V(x) = \frac{\hbar^2 q}{m}\delta(x)$, assume $x^{-1} \ll q \ll k_F$.

You may want to consider fermions in box of finite size L, and then take a limit $L \to \infty$, $N \to \infty$ such that $n = N/L = \text{const}$. Express your answer in terms of Fermi wavevector k_F.

Problem 5.2 (Hall conductivity — **4 points**). Consider 2D/3D electron gas with spectrum $\varepsilon_{\mathbf{p}} = \frac{\mathbf{p}^2}{2m}$ and negative charge $q = -|e|$ in crossed electric and magnetic fields $\mathbf{E} \perp \mathbf{H} \parallel \hat{z}$.

1. (**1 point**) Using Boltzmann equation in scattering time approximation calculate Hall conductivity σ_{xy} to order H. Express your answer in terms of Drude conductivity σ_0 and cyclotron frequency Ω.
2. (**1 point**) Calculate Hall resistivity ρ_{xy} to order H and the Hall coefficient $R_H = \rho_{xy}/H$.
3. (**1 point**) Calculate magnetoresistance $\Delta\rho_{xx} = \rho_{xx}(H) - \rho_{xx}(0)$. How would $\Delta\rho_{xx}$ change if electrons have anisotropic mass $m_x \neq m_y = m_z$?
4. (**1 point**) Measuring the transverse conductivity of a sample in the lab, assume that the contacts on both sides of the sample are not aligned properly, and therefore you measured a combination of Hall and longitudinal conductances. Propose a method to eliminate the longitudinal contamination in the measured Hall signal.

Problem 5.3 (Two band model— **2 points**)**.** Let's say there are two types of carriers with different concentrations and mobilities (i.e. different resistivity $\rho_{1,2}$ and Hall coefficients $R_{1,2}$). Compute resistivity of such two component system and write it down in standard form

$$\hat{\rho} = \begin{pmatrix} \rho_0 & -RH \\ RH & \rho_0 \end{pmatrix}$$

Determine ρ and R in terms of ρ_1, ρ_2, R_1 and R_2.

Problem 5.4 (Helicon — **3 points**)**.** There exists in metals a unique mode of propagation of electromagnetic waves (discovered by R. Bowers *et al*) that is intimately related to the Hall effect. Suppose we place a metal in a uniform magnetic field $\mathbf{H} = H\hat{z}$ (perpendicular to the surface of the metal) and shine circularly polarized light on its surface $\mathbf{E} = \mathrm{Re}\, E_0(\hat{x} + i\hat{y})e^{ikz - i\omega t}$ (light propagates along the direction of magnetic field).

1. (**1 point**) Derive relation between current $\mathbf{j} = \hat{\sigma}(\omega)\mathbf{E}$ and electric field inside the metal within Drude approximation.

   $$\frac{d\mathbf{p}}{dt} = -\frac{\mathbf{p}}{\tau} - |e|\mathbf{E} - \frac{|e|}{m}\left[\frac{\mathbf{p}}{c} \times \mathbf{H}\right], \qquad \mathbf{j} = -\frac{|e|n}{m}\mathbf{p}.$$

   Express your answer in terms of Drude conductivity σ_0 and cyclotron frequency ω_c.

2. (**1 point**) Using Maxwell equations find dispersion relation $k^2c^2 = \varepsilon(\omega)\omega^2$ for such waves, i.e. determine $\varepsilon(\omega)$. Express your answer in terms of plasma frequency ω_p and cyclotron frequency ω_c.

   (2a) Assume that $\omega_c\tau \ll 1$. At what frequencies light propagates inside the metal?

3. (**1 point**) Consider a very pure metal with resistivity $\rho \simeq 10^{-9}$ $\Omega\cdot$cm and typical carrier density $n \simeq 10^{23}$ cm^{-3} that we have placed in strong magnetic field of magnitude $B = 5$ T. Provide numerical estimates for typical values of cyclotron frequency ω_c, plasma frequency ω_p and mean–free time τ.

   (3a) Show that in the regime of large field $\omega_c\tau \gg 1$ and low frequency $\omega \ll \omega_c$, dispersion relation reduces to

   $$\omega = \frac{\omega_c}{\omega_p^2}k^2c^2.$$

   This low frequency wave is known as *helicon*, and it is observed in many metals.