Exam

due on 6/6/2021

Physics–172 / Applied Physics–272

Introduction to Solid State Physics

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Problem e.1 (Specific heat at constant pressure — **6 points**)**.** In the lab we always measure the specific heat at constant pressure c_P, while in class we only calculated the specific heat at constant volume c_V. Rationalize this process by showing that for electronic Fermi gas

$$\frac{c_P}{c_V} = 1 + A\left(\frac{T}{\epsilon_F}\right)^2 + \mathcal{O}\left(\frac{T}{\epsilon_F}\right)^4.$$

Compute factor A. Provide an expression for ϵ_F in your answer.

Problem e.2 (Kagome lattice — **6 points**)**.** In many materials atoms are arranged in so–called *kagome lattice* illustrated on Figure 1. It could be described as triangular lattice together with its two copies shifted by the half of primitive vectors. On the figure below white atoms shifted by $\frac{1}{2}\mathbf{a}_1$ produce triangular lattice of blue atoms, and shift by $\frac{1}{2}\mathbf{a}_2$ gives red atoms. Lattice size a is the distance between nearest neighbors, so that $|\mathbf{a}_1| = |\mathbf{a}_2| = 2a$.

1. (**4 points**) Find spectrum $\varepsilon_{\mathbf{k}}$ of electrons living on the kagome lattice in tight–binding approximation (only consider nearest–neighbor hopping).

$$\hat{H} = -t\sum_{\langle i,j\rangle} \hat{c}^{+}(\mathbf{r}_i)\hat{c}(\mathbf{r}_j).$$

2. (**2 points**) Imagine that each atom contributes 5/6 electrons, at what energy Fermi level is located? Is it a metal or an insulator? At what energy is Dirac cone located?

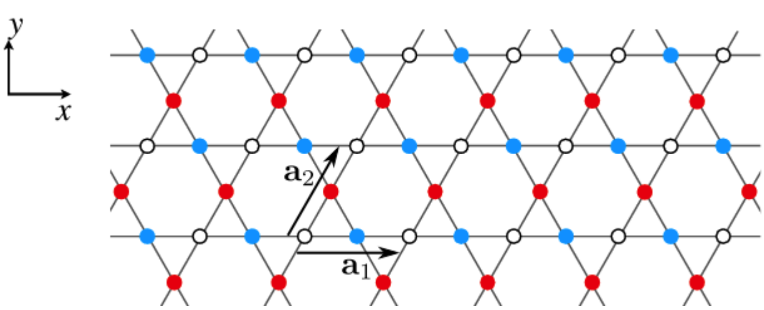

Figure 1: Primitive translation vectors $\mathbf{a}_1$, $\mathbf{a}_2$ of the kagome (triangular) lattice.

Problem e.3 (Graphene photodetector — **9 points**)**.** The goal of this question is to evaluate graphene performance as a bolometer–type photodetector. Graphene is a two-dimensional carbon monolayer in which low-energy electronic states are well described by a linear band dispersion $E(\mathbf{p}) = v|\mathbf{p}|$ for electrons and $E(\mathbf{p}) = -v|\mathbf{p}|$ for holes. The velocity value is $v = 10^6$ m/s.

Consider a graphene monolayer sample of size 1 micron by 1 micron that absorbs one photon of energy 2.7 eV (blue light). Taking initial temperature to be $T = 2$ K, find the change in electron temperature resulting from photon absorption. Assume that the photon energy is shared among the electron degrees of freedom and ignore the effects of electron–lattice cooling. Please provide your answer in kelvin.

1. (**4 points**) Assume that graphene is undoped, i.e. Fermi level is exactly at the Dirac point.
2. (**4 points**) Now suppose that a small number of carriers are added to the band at a total density of $n = 10^{12}$ cm^{-2}. Repeat the analysis of part 1 to find the change in electron temperature after a single photon absorption.

Could your answers be used as reasonable estimates for a real graphene? Explain (**1 point**).

Problem e.4 (Cluster classical Heisenberg model — **4 points**)**.** Consider a system of N classical spins $\mathbf{n}_i \in S^2$ that have all–to–all ferromagnetic ($J > 0$) interaction.

$$E = -\frac{J}{2N}\sum_{i\neq j}\mathbf{n}_i \cdot \mathbf{n}_j - \sum_{i=1}^{N}\mathbf{h}\cdot\mathbf{n}_i,$$

Find the Curie temperature T_c in the thermodynamic limit $N \to \infty$. You may choose to solve the problem exactly or within mean–field approximation.