Exam

due on 6/14/2023

Physics–172 / Applied Physics–272
Introduction to Solid State Physics
Spring quarter, 2023

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Problem e.1 (Nearly free electrons— **10 points**). Consider one–dimensional periodic potential of triangular form as small perturbation.

$$\hat{\mathcal{H}} = \frac{\hat{p}^2}{2m} + \frac{\hbar^2 q^2}{m} v_a(x), \qquad v_a(x) = \sum_{n=-\infty}^{\infty} \left[1 - \frac{4}{a}|x - na|\right] \theta\left(2 - \frac{4}{a}|x - na|\right).$$

How does small periodic potential change spectrum of free electrons (no calculation is needed to answer this part of the problem)? Find the width of energy band gaps Δ_n up to the first order in potential strength $q^2 \ll 1/a^2$.

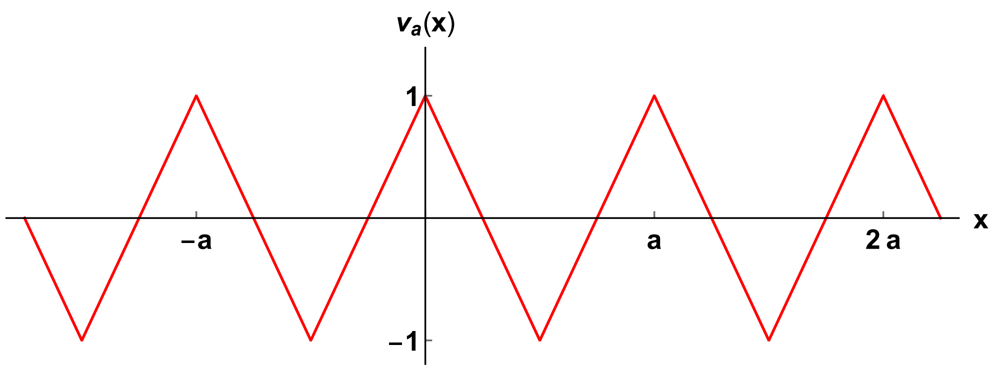

Figure 1: Triangle periodic potential.

Problem e.2 (Kagome lattice — **14 points**). In many materials atoms are arranged in so–called *kagome lattice* illustrated on Figure 2. It could be described as triangular lattice together with its two copies shifted by the half of primitive vectors. On the figure below white atoms shifted by $\frac{1}{2}\mathbf{a}_1$ produce triangular lattice of blue atoms, and shift by $\frac{1}{2}\mathbf{a}_2$ gives red atoms. Lattice size a is the distance between nearest neighbors, so that $|\mathbf{a}_1| = |\mathbf{a}_2| = 2a$.

1. (**8 points**) Find spectrum $\varepsilon_{\mathbf{k}}$ of electrons living on the kagome lattice in tight–binding approximation (only consider nearest–neighbor hopping).

$$\hat{H} = -t \sum_{\langle i,j \rangle} \hat{c}^{+}(\mathbf{r}_i)\hat{c}(\mathbf{r}_j), \quad t > 0.$$

2. (**6 points**) Imagine that each atom contributes 5/6 electrons, at what energy Fermi level is located? Is it a metal or an insulator? At what energy is Dirac node located?

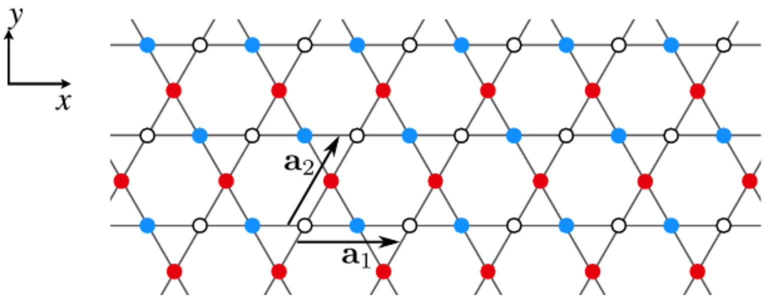

Figure 2: Primitive translation vectors $\mathbf{a}_1$, $\mathbf{a}_2$ of kagome (triangular) lattice.

Problem e.3 (Graphene photodetector — **18 points**). The goal of this question is to evaluate graphene performance as a bolometer–type photodetector. Graphene is a two-dimensional carbon monolayer in which low-energy electronic states are well described by a linear band dispersion $\varepsilon_{+,\mathbf{p}} = v|\mathbf{p}|$ for electrons and $\varepsilon_{-,\mathbf{p}} = -v|\mathbf{p}|$ for holes in each valley, where $v = 10^6$ m/s.

Consider a graphene monolayer sample of size 1 micron by 1 micron that absorbs one photon of energy 2.7 eV (blue light). Taking initial temperature to be $T = 2$ K, find the change in electron temperature resulting from photon absorption. Assume that the photon energy is shared among the electron degrees of freedom and ignore the effects of electron–lattice cooling. Please provide your answer in kelvin.

1. (**8 points**) Assume charge neutrality, i.e. electron and hole densities are the same $n_e = n_h$.

2. (**8 points**) Now suppose that a small number of carriers are added to the band at a total density of $n = 10^{12}$ cm^{-2}. Repeat the analysis of part 1 to find the change in electron temperature after a single photon absorption.

Could your answers be used as reasonable estimates for a real graphene? Explain (**2 points**).

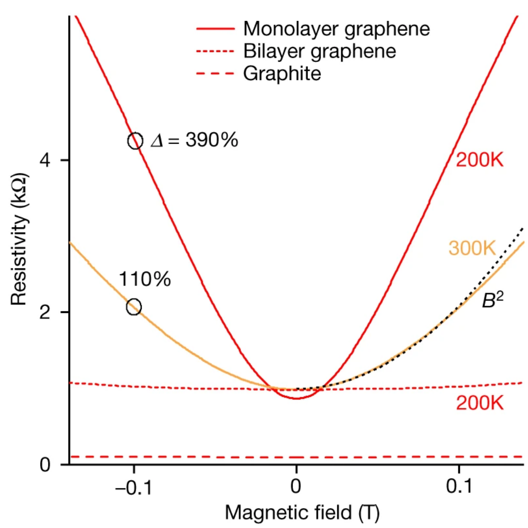

Figure 3: Resistivity of compensated graphene [1].

Problem e.4 (Graphene magnetoresistance — **18 points**)**.** In this problem we will describe electron magnetotransport in graphene using relaxation time approximation[1]. Reminder: $e < 0$.

$$\left(e\mathbf{E} + \frac{e}{c}\left[\mathbf{v_p} \times \mathbf{H}\right]\right) \cdot \frac{\partial f_\mathbf{p}}{\partial \mathbf{p}} = -\frac{f_\mathbf{p} - f_\mathbf{p}^{(0)}}{\tau}, \qquad \mathbf{j} = e\int_\mathbf{p} \mathbf{v_p} f_\mathbf{p}, \qquad \mathbf{v_p} = \frac{\partial \varepsilon_\mathbf{p}}{\partial \mathbf{p}}.$$

1. (**9 points**) Derive an expression for conductivity σ of graphene in weak perpendicular magnetic field within τ–approximation at charge neutrality and room–like temperature.

2. (**9 points**) Compare your formulas to experimental data on Fig. 3. What value of $\Gamma_\tau = \hbar/\tau$ is needed to have magnetoresistance $\Delta = 110$ % at $T = 300$ K and $H = 100$ mT? Provide your answer in meV. Data is available online.

$$\Delta(H) \equiv \frac{R_{xx}(H) - R_{xx}(0)}{R_{xx}(0)}.$$

References

[1] Na Xin, James Lourembam, Piranavan Kumaravadivel, A. E. Kazantsev, Zefei Wu, Ciaran Mullan, Julien Barrier, Alexandra A. Geim, I. V. Grigorieva, Artem Mishchenko, et al. Giant magnetoresistance of dirac plasma in high-mobility graphene. *Nature*, 616(7956):270–274, 2023.

[1] Berry's curvature contribution is ignored for simplicity.