2 Assignment on «Numerical computations»

Problem 2.1. Solve Korteweg de-Vries equation

$$u_t + u_{xxx} + 6uu_x = 0$$

with two-soliton initial condition

$$u|_{t=0} = \operatorname{ch}^{-2}\left(\frac{x-5}{\sqrt{2}}\right) + \frac{3}{2}\operatorname{ch}^{-2}\left(\frac{x+5}{2/\sqrt{3}}\right)$$

and periodic boundary conditions (for u, u_x and u_{xx}). Animate the solution. Do solitons interact or pass through each other without a change?

Problem 2.2. Consider the following integral as a function of parameter *a*.

$$J(a) = \int_0^{10} \frac{dx}{ch[(x-a)^2]}.$$

From analytical reasons it is obvious that J(a) has maximum at a = 5. Try to obtain such result using NMaximize with 10-digits precision. Notice that due to very small derivate near the maximum, precision of maximum point a_{max} is much smaller than precision of maximum value $J(a_{\text{max}})$. Construct a function that finds point of maximum with given n-digits precision.