4   Assignment on «Eigenproblems»

Problem 4.1. Consider a Pöschl–Teller potential.

$$-\psi''(x) + \psi(x) - \frac{3}{2}\frac{\psi(x)}{\operatorname{ch}^2 x} = \lambda\psi(x), \quad \psi(\pm\infty) = 0;$$

It is easy to see that it has two bounded states

$$\psi_0(x) = \sqrt{\frac{3}{8}}\frac{1}{\operatorname{ch}^2 \frac{x}{2}}, \quad \psi_1(x) = \sqrt{\frac{3}{4}}\frac{\operatorname{sh}\frac{x}{2}}{\operatorname{ch}^2 \frac{x}{2}}$$

with energies $\lambda_0 = 0$, $\lambda_1 = \frac{3}{4}$. Let's practice numerical methods of finding its eigensystem.

1. Can `DEigensystem` find these solutions?

2. Solve eigenproblem using `NDEigensystem`.

3. Solve eigenproblem using `ParametricNDSolve`.

4. Solve eigenproblem in finite matrix representation (using Hermite basis).

Compare results with each other and with an exact ones.